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SYNTACTIC RULES FOR FORMAL LOGIC

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TECHNICAL NOTE

The draft following this note forms part of a projected pamphlet summarising those aspects of logic which are most useful to students who must evaluate fairly complex logical arguments, but who do not want prolonged training in logic. The first part of the pampghlet would deal with informal logic: this part covers formal logic.

I observe that systems of logic can be most readily understood if they are treated as languages. For this reason, both syllogistic logic and the propositional calculus are developed here by means of intuitively acceptable syntactic rules. It is shown that the rules needed to develop an important part of propositional logic are isomorphic with syllogistic principles, at least when the principle of subalternation is amended to require the existential assumption which is not made explicitly in traditional treatments. Many textbooks compare logic and Boolean algebra, but my notion of using the duality of interpretation of certain rules of natural deduction as a teaching device appears to be original.

The rules enable a student to formalize valid inferences by constructing suppositional proofs. In this way it is possible to sidestep consideration of the paradoxes associated with the definition of implication. To continue to save the student from this source of bafflement, it is necessary to avoid the use of truth tables. Therefore the decision procedure given for propositional logic is the one involving reduction to conjunctive normal form.

The predicate calculus is not treated because it is much more complex but not much more useful to my intended readers than syllogistic logic,

Traditional logicians will note that the rules stated first incorporate only devices traditionally used in syllogistic logic, so that they cannot be used to derive invalid forms. The first two rules symbolise the main valid moods of the first figure, Rule 1 corresponding to Barbara and Celarent, Rule 2 to Darii and Ferio. The next three rules correspond to immediate inferences which enable syllogisms in other figures to be reduced to the first figure; thus Rule 3 embodies the principle of subalternation--permitting the weakened moods to be derived also; Rule 4 corresponds to the conversion of an I proposition or to the contraposition of an O proposition; and Rule 5 to the contraposition of an A proposition or, with the additional application of the special Rule 6, to the conversion of an E proposition.

THE RULES OF REASONING

Class Logic

Considering only its forms, a language is nothing but a vocabulary, that is to say, a set of distinguishable items, and a grammar, that is to say, a set of rules defining the ways in which the items may be combined, thus forming sentences. Language is useful because its forms have meaning: sentences are interpretable as relating to real or imaginary states of affairs. If a declarative sentence is observed to correspond to a real state of affairs it is said to be true;if it is observed not to correspond to a real state of affairs it is false; if the relevant state of affairs could be observed but has not been, the sentence is only possibly true; if the state of affairs could not, even in principle, be observed, then the sentence is, strictly speaking, meaningless.

Logic is a language--but a rather special one. The important difference between the grammar of everyday language and the grammar of logic is that the latter is a truth-preserving grammar; this means that the rules of logic are so restricted that it is impossible to construct a false, only possible true, or meaningless sentence from true sentences. The grammatical rules governing a natural language impose no such restriction. For example, given a positive sentence, there is a rule of English grammar which permits its transformation into a new sentence by inserting the word "not". Thus it is grammatically correct to say "all men are mortals, and all men are not mortals." That utterance is, however, logically unacceptable because there is no rule in logic permitting the simple transformation of a positive statement into a negative one. If there were such a rule it would reduce logic to an ordinary language in which one can talk nonsense and still be grammatically correct.

Logic is useful, then, because it enables us to establish true statements by making inferences. An inference consists of one or more statements, called assumptions, from which another statement, called the conclusion, is derived. An inference is valid if the conclusion is derived only by the correct application of the rules of a logical system. The truth of the conclusion is guaranteed if, but only if, the assumptions are true and the inferences valid. The following six rules, it will be shown, suffice to define a logical system called syllogistic:

$$(1)\ \begin{array}{c} x/y \\ y/z \\ \hline x/z \end{array} \qquad (2)\ \begin{array}{c} x.y \\ y/z \\ \hline x.z \end{array} \qquad (3)\ \begin{array}{c} x/y \\ \hline x.y \end{array} \qquad (4)\ \begin{array}{c} x.y \\ \hline y.x \end{array} \qquad (5)\ \begin{array}{c} x/y \\ \hline -y/-x \end{array} \qquad (6)\ \begin{array}{c} --x \\ \hline x \end{array}$$

These rules are to be understood as permitting any combination(s) of symbols, called an expression, which occur(s) above the black line to be replaced by the expression below the line.

Let us now interpret the expressions as sentences by assigning meanings to the symbols. The letters x, y, z, which are chosen arbitrarily, stand for classes of things. A class is the collection of all things having some specified characteristic(s) in common; for example, "musicians" is the name of a class all members of which are characterized by the fact that they compose or play music. "Beethoven" is the name of a class although he is dead and his class has only one member. For the moment any class must be presumed to have at least one member. Each letter in the rules above may therefore be replaced by the name of a class.

The symbol / means "are all". Thus the expression x/y could stand for

men are all mortals

which in more usual English is

all men are mortal,

a class name wherever it occurs--in isolation or combined in an expression with other class names.

So that you can see how reasonable the rules are, here are some possible interpretations of them.

	x/y	assuming that	all musicians are long-haired people
	y/z	and that	all long-haired people are music lovers
(1)	x/z		all musicians are music lovers

Here is another example with a class of only one member:

if	Beethoven was a long-haired person
and	all long-haired persons are music lovers
then	Beethoven was a music lover

	x.y		some professors are knowledgeable people
	y/z		all knowledgeable people are bores
(2)	x.z	therefore	some professors are bores

Here is Rule 2 with substitutions making it applicable to two negative classes, non-residents and non-taxpayers:

	x.-y	from the facts that	some foreigners are non-residents
	-y/-z	and that	no non-residents are local taxpayers*
	x.-z	it follows that	some foreigners are not local taxpayers
	x/y	let us say	all women are bad drivers
(3)	x.y	then	some women are bad drivers
	x.y	given that	some women are bad drivers
(4)	y.x		some bad drivers are women
	x/y		all logicians are wise people
(5)	-y/-x	in other words	no unwise people are logicians
	--x	to say that	non-non-logicians exist
(6)	x	is to say that	logicians exist
	y/--x		no psychiatrists are non-medical doctors**
	y/x	means	all psychiatrists are medical doctors

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* i.e. all non-residents are non-local-taxpayers

** i.e. all psychiatrists are non-non-medical-doctors

We have now all the apparatus necessary to enumerate all possible forms of syllogisms, as we call valid inferences about classes from pairs of assumptions having the following forms:

x/y	meaning "all x are y,"
x/-y	meaning "no x are y,"
x.y	meaning "some x are y,"
x.-y	meaning "some x are not y."

(Remember that x or y may represent negative class names.)

There are only 24 syllogistic forms; they are set out in the table below, together with their derivations. In the table, the letters S, M, P are used to stand for terms used in traditional logic, in which the first and second classes named in the conclusion are called the Subject term and the Predicate term respectively, and the class common to the two assumptions is called the Middle term. (As an exercise, check the reasonableness of these inferences by substituting Sculptors, Musicians and Poets, or Scientists, Mathematicians and Philosophers for S, M, P.)

Table of Syllogisms

Assumptions		Derivation*				Conclusion
S/M	M/P					(1) S/P
S/M	M/P	(1) S/P				(3) S.P
S/M	M/-P					(1) S/-P
S/M	M/-P	(1) S/-P				(3) S.-P
S.M	M/P					(2) S.P
S.M	M/-P					(2) S.-P
S/M	P/-M	(5) --M/-P	(6) M/-P			(1) S/-P
S/M	P/-M	(5) --M/-P	(6) M/-P	(1) S/-P		(3) S.-P
S/-M	P/M	(5) -M/-P				(1) S/-P
S/-M	P/M	(5) -M/-P	(1) S/-P			(3) S.-P
S.M	P/-M	(5) --M/-P	(6) M/-P			(2) S.-P
S.-M	P/M	(5) -M/-P				(2) S.-P
M/S	M/P	(3) M.S	(4) S.M			(2) S.P
M/S	M/-P	(3) M.S	(4) S.M			(2) S.-P
M/S	M.P	(4) P.M	(2) P.S			(4) S.P
M.S	M/P	(4) S.M				(2) S.P
M/S	M.-P	(4) -P.M	(2) -P.S			(4) S.-P
M.S	M/-P	(4) S.M				(2) S.-P
M/S	P/M	(1) P/S	(3) P.S			(4) S.P
M/-S	P/M	(1) P/-S	(5) --S/-P			(6) S/-P
M/-S	P/M	(1) P/-S	(5) --S/-P	(6) S/-P		(3) S.-P
M/S	P.M	(2) P.S				(3) S.P
M/S	P/-M	(5) --M/-P	(6) M/-P	(3) M.S	(4) S.M	(2) S.-P
M.S	P/-M	(5) --M/-P	(6) M/-P	(3) S.M		(2) S.-P

*Each number refers to the rule used to derive the succeeding expression.

It is easy to check whether an argument symbolized in syllogistic form is acceptable. A valid syllogism must satisfy the following criteria:

1. The midlle term must appear at least once to the left of a / symbol or to the right of a . symbol.
2. The . symbol can appear in only one assumption; and if it does it must appear in the conclusion.
3. If the - symbol appears before the predicate it must appear before both occurrences of the middle term or before neither of them; and if the - symbol appears an odd number of times in the assumptions it must appear in the conclusion (for such an instance look back to the second example given for Rule 2: the example corresponds to the syllogistic form S.M and M/-P, therefore S.-P).

The syllogisms symbolised in the table constitute nearly all the valid forms of inference that were studied by Western logicians until a century ago. However further valid inferences can be derived by substituting complementary classes for positive ones, as was done in the examples showing how the rules could be used. It is also possible to apply Rule 1 repeatedly thus:

S/M	e.g. assuming that	all Sculptors are Musicians
$\underline{M/P}$	and that	all Musicians are Poets
S/P		all Sculptors are Poets
$\underline{P/E}$	and assuming that	all Poets are Egg-heads
S/E		all Sculptors are Egg-heads

Still further forms of inference about classes may be made by adding to the rules. For instance, we may add:

(7) $\dfrac{x, y}{y}$ e.g. some birds are ravens therefore there are ravens

where the isolated letter y means "y exists".

A shortcoming of the traditional system presented above is that it makes the tacit assumption that there is at least one member of each class. Suppose we are tempted to make inferences about the class of unicorns. This is an empty class, because there never have been any beasts characterized by looking like horned white ponies. It is therefore not valid to use Rule 3 to derive a conclusion such as "some unicorns are charming beasts," which could not be true unless there were at least one unicorn. To make sure that Rule 3 is not used incorrectly, it must be removed thus:

	x/y	e.g.	all cats are charming beasts
	$\underline{x}$		cats exist
(3')	x.y	so	some cats (possibly all) are charming beasts

The result of applying Rule 7 to the conclusion of Rule 3' may be conveniently summarized in a new rule:

	x/y	e.g.	all cats are charming beasts
	$\underline{x}$		cats exist
(8)	y	so	charming beasts exist

Another rule which is also valid for classes (more precisely, for empty classes) is:

	x/y	e.g.	all snarks are boojums
	$\underline{-y}$		there are no boojums
(9)	-x	therefore	there are no snarks

Furthermore, we can introduce a new symbol V to mean "... or ... (or possibly both) exist". Three more rules can now be stated:

	$\underline{x}$	e.g. if it is true that	ravens exist
(10)	x V y	it may be fatuous to say so but it is nevertheless true to say that	ravens or unicorns (or any other things you name) exist

	x V y	e.g. assuming it is really true that	ravens or unicorns exist
	$\underline{-y}$	and that	there are no unicorns
(11)	x		ravens must exist

(12) $\dfrac{x \text{ V } y}{y \text{ V } x}$ e.g. saying "ravens or unicorns exist"
is the same as saying "unicorns or ravens exist"

Propositional Logic

Now comes a surprise. All the rules given above (with Rule 3' substituted for Rule 3) can be given a completely different interpretation so that they apply not to classes but to declarative sentences, called propositions. In propositional logic a letter stands for a proposition, which may consist of a combination of further propositions. A letter standing on its own simply asserts that the proposition for which it stands is true.

The symbol / now means "implies". For example, x/y could stand for

the cat is away implies the mice will play,

which is usually expressed in English as

if the cat's away the mice will play.

The symbol . means "and". For instance, x.y could stand for

your house is on fire and your children are home.

The symbol - means "it is not the case that". Thus -x could denote

it is not the case that I love thee, Doctor Fell,

which is better rendered

I do not love thee, Doctor Fell.

The symbol V means "it is the case that . . . or . . . , or possibly both". This interpretation can best be illustrated by showing what Rules 10 and 11 could now mean.

(10) $\dfrac{x}{x \text{ V } y}$ e.g. my name is McLaughlin
so my name is McLaughlin or I'm a Dutchman

(11) $\begin{array}{l} x \text{ V } y \\ \underline{\quad -y} \\ x \end{array}$ e.g. my name is McLaughlin or I'm a Dutchman
I am not a Dutchman
so my name is McLaughlin

To illustrate the duality of the rules, a valid form of inference will now be set out symbolically, followed by two possible interpretations, one syllogistic, the other propositional. Notice that the lines are numbered with small roman numerals. Following each line there is a parenthesis giving the number of the rule and line(s) from which it was derived.

i	M/S	(assumption)
ii	M	(assumption)
iii	P/-M	(assumption
iv	M.S	(3'; i,ii)
v	S.M	(4; iv)
vi	--M/-P	(5; iii)
vii	M/-P	(6; vi)
	S.-P	(2; v, vii)

i All Musicians are Sculptors

ii Musicians exist

iii No Poets are Musicians

iv Some (at least one, possibly all) Musicians are Sculptors

v Some Sculptors are Musicians

vi No non-non-Musicians are Poets

vii No Musicians are Poets

Some Sculptors are not Poets

i If Mark makes money then Sister Susy sulks

ii Mark makes money

iii If Peter plays poker then it is not the case that Mark makes money

iv Mark makes money and Sister Susy sulks

v Sister Susy sulks and Mark makes money

vi If it is not the case that it is not the case that Mark makes money, then it is not the case that Peter plays poker

vii If Mark makes money, then it is not the case that Peter plays poker

Sister Susy sulks and it is not the case that Peter plays poker

It is possible to find even more forms of inference which are valid in both syllogistic and propositional logic. However, the two systems are not identical in structure. Some rules hold only for propositional logic. For instance

$$\begin{array}{c} x \\ \underline{\quad y \quad} \\ x.y \end{array}$$

is always true when interpreted to mean that two separate sentences remain equally true when they are joined by the word "and". The rule is not valid if taken to mean that any two classes must have members in common--thus if cats exist and dogs exist it does not follow that some cats are dogs!

Just as Rules 1 through 6 are sufficient to derive all valid syllogistic inferences, a list of rules can be set up sufficient to derive all valid propositional inferences. In the following list each rule is labelled by capital letters standing for its title, which is intended to be self-explanatory:

AI (And Introduction)

$$\begin{array}{c} x \\ \underline{\quad y \quad} \\ (x.y) \end{array}$$

AE (And Elimination)

$$\begin{array}{c} \underline{(x.y)} \\ y \end{array}$$

AT (And Transposition)

$$\begin{array}{c} \underline{(x.y)} \\ (y.x) \end{array}$$

OI (Or Introduction)

$$\begin{array}{c} \underline{\quad x \quad} \\ (x \vee y) \end{array}$$

OE (Or Elimination)

$$\begin{array}{c} (x \vee y) \\ \underline{\quad -y} \\ x \end{array}$$

OT (Or Transposition)

$$\begin{array}{c} \underline{(x \vee y)} \\ (y \vee x) \end{array}$$

DII (Direct If Introduction)

$$\begin{array}{ll} x_1 & \text{(assumption)} \\ x_2 & \text{"} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ x_{n-1} & \text{"} \\ \underline{x_n} & \text{"} \\ \hline \quad y & \text{(derived by rules)} \\ \hline \multicolumn{2}{l}{(x_1.x_2. \ \ldots \ .x_{n-1})/(x_n/y)} \end{array}$$

III (Indirect If Introduction)

$$\begin{array}{ll} x_1 & \text{(assumption)} \\ x_2 & \text{"} \\ \cdot & \cdot \\ \cdot & \cdot \\ x_{n-1} & \text{"} \\ x_n & \text{"} \\ \underline{-y} & \underline{\text{(denial of conclusion)}} \\ \multicolumn{2}{l}{\underline{\text{contradiction (derived by rules)}}} \\ \multicolumn{2}{l}{(x_1.x_2. \ \ldots \ .x_{n-1})/(x_n/y)} \end{array}$$

IE (If Elimination)

$$\begin{array}{c} (x/y) \\ \underline{\quad x \quad} \\ y \end{array}$$

It will be noticed that And Elimination (AE) is our old Rule 7; And Transposition (AT) is Rule 4; Or Introduction (OI), Or Elimination (OE), and Or Transposition (OT) are Rules 10, 11 and 12 respectively; If Elimination (IE) is Rule 8. The new rule of And Introduction (AI) has just been discussed. Direct If Introduction (DII) and Indirect If Introduction (III) are new rules which do not apply to classes; they will now be explained in detail.

Direct If Introduction (DII) is usually known as the method of Conditional Proof. It states that when the application of the other rules to n different assumptions leads to the derivation of the line y, this directly proves a conclusion to the following effect:

Supposing n-1 of the assumptions are true, it follows that,

if the nth assumption is also true, then y is true.

The use of DII is easier to grasp from an example:

1	(x/y)	(assumption)	assuming that if I sing you laugh
2	(y/z)	(assumption)	and that if you laugh I blush
3	x	(assumption)	and that I sing,
4	y	(IE; 1,3)	then you laugh
5	z	(IE; 2,4)	so I blush
(T1)	((x/y).(y/z))/(x/z)	(DII;1,2,3,5)	therefore supposing that if I sing you laugh and that if you laugh I blush it follows that if I sing I blush.

A conclusion derived by either method of If Introduction is called a theorem. Once proved, a theorem may be used in exactly the same way as a rule; in fact, Theorem T1 just established is identical with the old Rule 1.

Some theorems cannot be proved directly. It is then necessary to resort to the strategy provided by Indirect If Introduction (III), usually called the method of Indirect Proof, or reductio ad absurduum. III requires you to decide, before beginning the derivation, what the proposition y should be in the conclusion. You then add -y to the assumptions; that is, you deny the expected conclusion for the purposes of argument. The object is now to use the rules, other than the rules of proof, to derive an expression which contradicts any of the assumptions or any derived expression. When this occurs, it shows that an absurd set of assumptions has been chosen, because two contradictory expressions cannot both be true. Presuming that a check has been made to insure that the other assumptions do not lead to contradiction, the cause of the trouble must be -y. It follows that y is true. Thus the same type of conclusion as that given by DII is reached. Here is an example of the use of III, with a possible interpretation in ordinary English:

1	(x/y)	(assumption)	If my car works then it must contain gas.
2	(x.-y)	(denial of conclusion)	Suppose for the sake of argument that it is true to say both that my car works and that it is out of gas;
3	-y	(AE; 2)	then my car is out of gas.
4	(-y.x)	(AT; 2)	
5	x	(AE; 4)	But my car works,
6	y	(IE;contradicts 3)	so it follows from my first assertion that my car contains gas, which contradicts the supposition we first made.
(T2)	(x/y)/-(x.-y)	(III; 1,-2)	So I conclude that, as my car contains gas if it works, then it cannot be correct to suppose that my car works when it is out of gas.

More theorems are proved in the Appendix.

Because a theorem is written on a single line, it is necessary to bracket the propositions to make their relationships clear. The correct use of brackets may be stated in two further rules, which are best expressed in words:

Bracket Introduction (BI): Any assumed or derived expression may be substituted for a letter in any rule or theorem, but if the expression includes more than one occurrence of a letter then the expression must be enclosed in brackets; furthermore, brackets may be introduced at will in any string from which they may be eliminated.

Bracket Elimination (BE): Within any string of letters (including letters immediately preceded by the - symbol) separated only by . symbols and brackets or only by V symbols and brackets, the brackets may be eliminated: furthermore, there is no need to retain brackets around a complete expression.

To illustrate both of these rules, let the expression p/q be substituted for x and pV(q/-((p.-q).r)) for y in the rule for And Elimination. Then instead of

$$\frac{(x.y)}{y} \qquad \text{we have} \qquad \frac{(p/q).(pV(q/-((p.-q).r)))}{pV(q/-((p.-q).r))}$$

Notice that in the first line each of the substitute propositions have had to be enclosed in brackets, but that brackets have not been retained around the complete expressions in either the first or second lines.

Notice also that the conclusion

pV(q/-((p.-q).r))

could be written (according to BE) thus

pV(q/-(p.-q.r))

and that expression could be rewritten (according to BI) thus

pV(q/-(p.(-q.r)))

because all three expressions mean the same thing; but removing either of the outer pairs of brackets is not permitted by the rules (because that would change the meaning of the expression or make it ambiguous, and the rules are framed to make that kind of change impossible).

To check the soundness of an inference it should be translated into logical symbolism, the economy of which makes it much easier to grasp relationships between propositions. Making such a translation is quite an art because English has a great many equivalents for the connective symbols used in logic. The following list of equivalent phrases will help in symbolizing inferences. As an exercise, complete the phrases by substituting for the letters propositions such as those suggested at the head of the list.

x	Pat likes you; John is in New York; etc.	
y	You like Pat; John is in London, etc.	
z	The earth is not flat; etc.	
-x	It is not the case that Pat likes you; etc.	
-y	You do not like Pat; etc.	
-z	The earth is flat; etc.	
x.y	x and y	
	x; y, too	
	x; however, y	
x V y	x or y, or possibly both	
	x or y (<u>WARNING</u>: this phrase is ambiguous--it could mean "x or y, but not both")	
x V y.-(x.y)	x or y, but not both	
	x or y (<u>WARNING</u>: this phrase is ambiguous--it could mean "x or y, or possibly both")	
	x, alternatively y	
x/y	if x, then y	
	x, therefore y	y, because x
	when x, y	y, whenever x
	x implies y	y, on condition that x
	x is a sufficient condition for y	
	y is necessary condition for x	
x/-y	if x, then y is not the case	
	x unless y	unless y, x
(x/y).(y/x)	x if and only if y (often abbreviated to "x iff y")	
	x is a necessary and sufficient condition for y	
	x is equivalent to y	

The last expression on the list can be more conveniently symbolized by x = y. To add the = symbol to the system requires these extra rules:

EI (Equivalence Introduction)

$$\frac{\begin{array}{c} x/y \\ y/x \end{array}}{x = y}$$

EE (Equivalence Elimination)

$$\frac{x = y}{x/y}$$

ET (Equivalence Transposition)

$$\frac{x = y}{y = x}$$

Here now is a method for deciding whether a given expression is valid or not, without deriving it from the rules. This decision procedure depends upon two facts. First, an expression must be valid if it is made up by connecting with . symbols a string of expressions each one of which is valid. Second, any string of propositions connected by V symbols must be valid if two of the propositions are contradictory. For example, it must be true to say "I am in New York or I am not in New York . . ."; adding ". . . or the moon is made of green cheese" (or any other proposition) will still result in a true expression. The decision procedure therefore consists in using the rules to reduce the given expression to the form

$$x_1 . x_2 . x_3 . \ldots$$

where each expression x_i is of the form

$$p_a \vee p_b \vee p_c \vee \ldots$$

If inspection shows that each x_i contains at least one pair of propositions of the form p_i, $-p_i$, then the reduced expression must be valid; therefore the equivalent original expression must be valid.

The decision procedure can be carried out quite mechanically using the following equivalences, which are derived in the appendix:

(T19)	$--x = x$
(T20)	$-(xVy) = (-x.-y)$
(T21)	$-(x.y) = (-xV-y)$
(T22)	$(x/y) = (-xVy)$
(T23)	$(x.(yVz)) = ((x.y)V(x.z))$
(T24)	$(xV(y.z)) = ((xVy).(xVz))$

The best course is to use T22 to remove any / symbols from the original expression; then use T20 and T21 to move - symbols from the outside to the inside of brackets, eliminating double symbols with T19; then use T23 and T24 to bring . symbols outside any brackets in which they occur. Lastly use the rule of Bracket Elimination to get rid of brackets on either side of V symbols.

This all sounds much more difficult than it really is. Here is an example in which we test an expression that might stand for "Supposing that if it rains then, if I have forgotten my umbrella, I will get wet, then if it rains I will get wet":

(r/(f/w))/(r/w)	
-rV(-fVw)V(-rVw)	(T20 used four times)
-rV-fVwV-rVw	(BE)

Because no letter appears in both positive and negated forms in the last line it must be concluded that the expression is not a valid form of inference--which may not have been immediately obvious from either the English version or its original symbolic representation.

Finally, here is another example of the decision procedure, showing that T2 really is a valid form of inference, because in each final V-connected string of propositions a contradiction occurs, as marked.

(x/y) / -(x.-y)	(T2; to be tested)
-(-xVy) V -(x.-y)	(T22, T22)
(--x.-y) V (-xV--y)	(T20, T21)
(x.-y) V (-xVy)	(T19)
(-xVy) V (x.-y)	(OT)
((-xVy)Vx) . ((-xVy)V-y)	(T24)
(-xVyVx) . (-xVyV-y)	(BE)

APPENDIX

It would be a good exercise to derive Theorems 19 through 24, given above . In case you have difficulty in doing this--or want to satisfy yourself without effort that the theorems are valid--here are all the necessary proofs. We begin with half a dozen theorems which will be needed in later proofs.

	1	--x	(assumption)
	2	-x	(denial of conclusion)
	3	-x	(2; contradicts 1)
(T3)		--x/x	(III)

Analogously to T3 we can prove

(T4)		x/--x

	1	xVx	(assumption)
	2	-x	(denial)
	3	x	(OE; 1, 2; contradicts 2)
(T5)		(xVx)/x	(III)

	1	yVz	(assumption)
	2	-((x.y)V(x.z))	(denial)
	3	-(yVz)	(AE, AE; 2; contradicts 1)
(T6)		(yVz)/((x.y)V(x.z))	(III)

	1	-(xVy)	(assumption)
	2	x	(denial)
	3	xVy	(OI; 2; contradicts 1)
(T7)		-(xVy)/-x	(III)

Analogously to T7 we can prove

(T8)		-(xVy)/-y

Using AI to combine the conclusions of T7 and T8 we derive

(T9)		-(xVy) /(-x,-y)

	1	-(x.y)	(assumption)
	2	-(-xV-y)	(denial)
	3	--x.--y	(T9; 2)
	4	x.y	(T3, T3; 3; contradicts 1)
(T10)		-(x.y)/(-xV-y)	(III)

The next two theorems could be derived by III but direct proofs are more satisfying so these will be used from now on.

	1	-x.-y	(assumption)
	2	--(-x.-y)	(T4; 1)
	3	-(--xV--y)	(T10; 2)
	4	-(xVy)	(T3, T3; 3)
(T11)		(-x.-y)/-(xVy)	(DII)
	1	-xV-y	(assumption)
	2	--(-xV-y)	(T4; 1)
	3	-(--x.--y)	(T9; 2)
	4	-(x.y)	(T3, T3; 3)
(T12)		(-xV-y)/-(x.y)	(DII)
	1	-xVy	(assumption
	2	x	(assumption)
	3	yV-x	(OT; 1)
	4	y	(OE; 3, 2)
(T13)		(-xVy)/(x/y)	(DII)
	1	x/y	(assumption)
	2	-(x.-y)	(T2; 1)
	3	-xV--y	(T10; 2)
	4	-xVy	(T3; 3)
(T14)		(x/y)/(-xVy)	(DII)

In the last example of the text, Theorem 2 was tested by the decision procedure which involved the use of T14. If you think that this produced a circular argument because T2 is used in the proof just given for T14, you should find another derivation of T14. There are at least two indirect proofs, one using only the original rules, the other requiring T13.

	1	(x.y)V(x.z)	(assumption)
	2	xVx	(AE, AE; 1)
	3	x	(T5; 2)
	4	yVz	(AE, AE; 1)
	5	x.(yVz)	(AI; 3, 4)
(T15)		((x.y)V(x.z))/(x.(yVz))	(DII)
	1	x.(yVz)	(assumption)
	2	yVz	(AE; 1)
	3	(x.y)V(x.z)	(T6; 2)
(T16)		(x.(yVz))/((x.y)V(x.z))	(DII)

	1	xV(y.z)	(assumption)
	2	xVy	(AE; 1)
	3	xVz	(AE; 1)
	4	(xVy).(xVz)	(AI; 2, 3)
(T17)		(xV(y.z))/((xVy).(xVz))	(DII)
	1	(xVy).(xVz)	(assumption)
	2	xVz	(AE; 1)
	3	(y.x)V(y.z)	(T6; 2)
	4	xV(y.z)	(AE; 3)
(T18)		((xVy).(xVz))/(xV(y.z))	(DII)
	1	--x/x	(T3)
	2	x/--x	(T4)
(T19)		--x=x	(EI)

By a similar use of Equivalence Introduction, we can also derive T20 from T9 and T11; T21 from T10 and T12; T22 from T13 and T14; T23 from T15 and T16; and T24 from T17 and T18.